9-27-2018

Weighted Composition Operators on Spaces of Analytic Functions: A Survey

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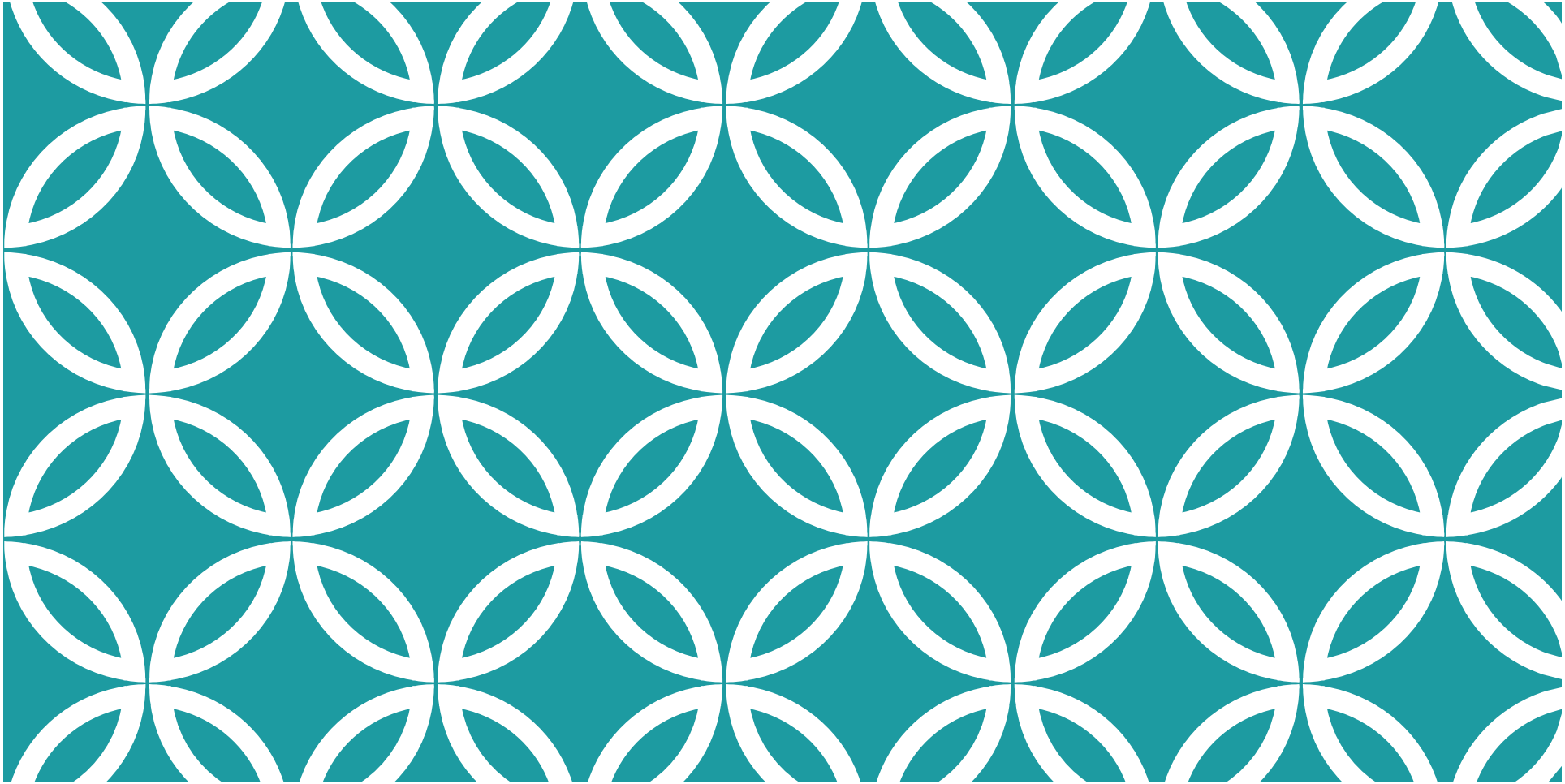


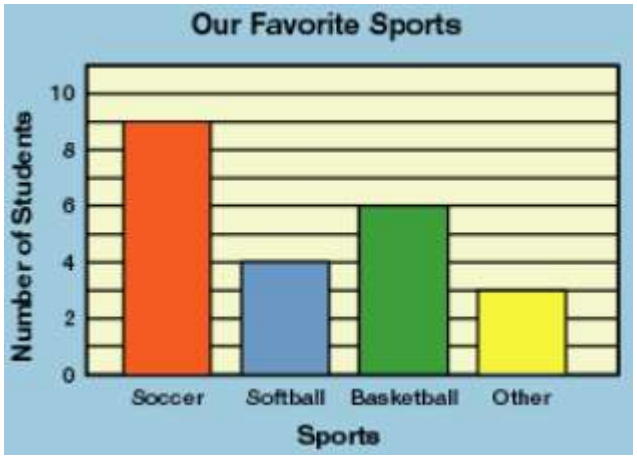
Weighted Composition Operators On Spaces of Analytic Functions

A Survey

OBJECTS STUDIED
In Math

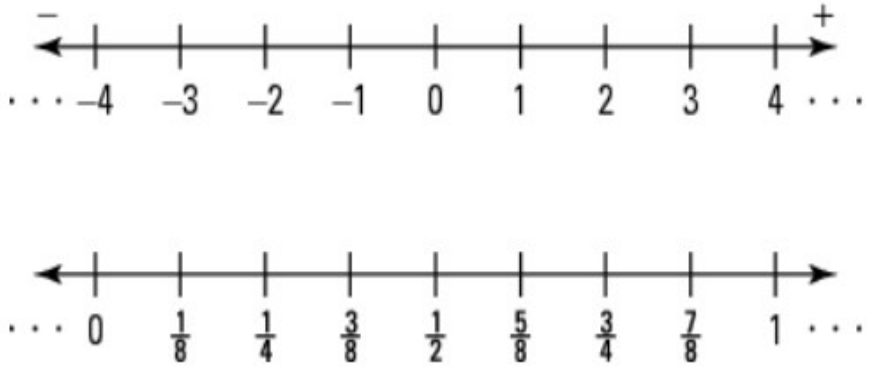
Shapes

Data

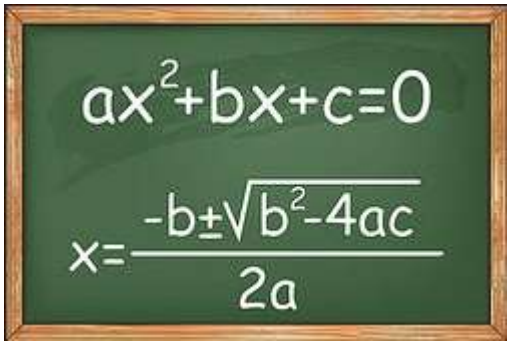


CONTINUED

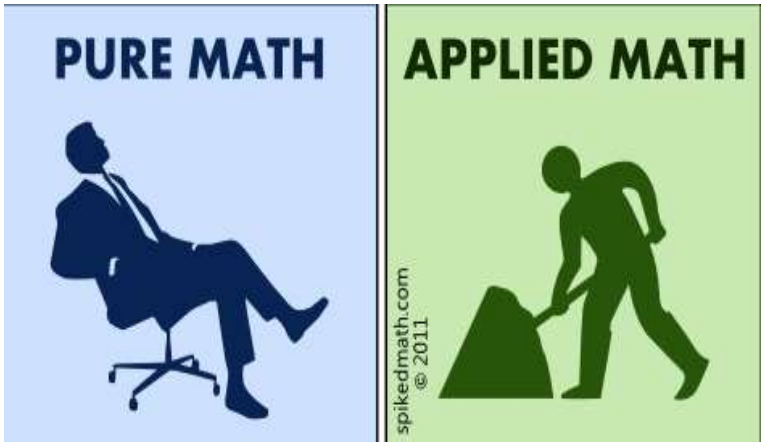
Numbers



Letters/Equations



USING ABSTRACTION

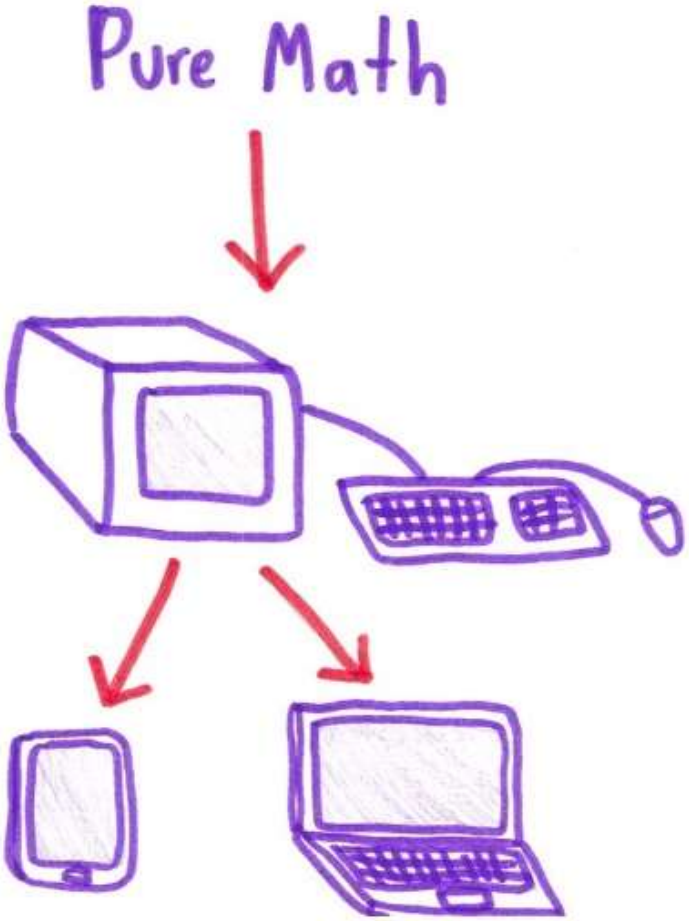


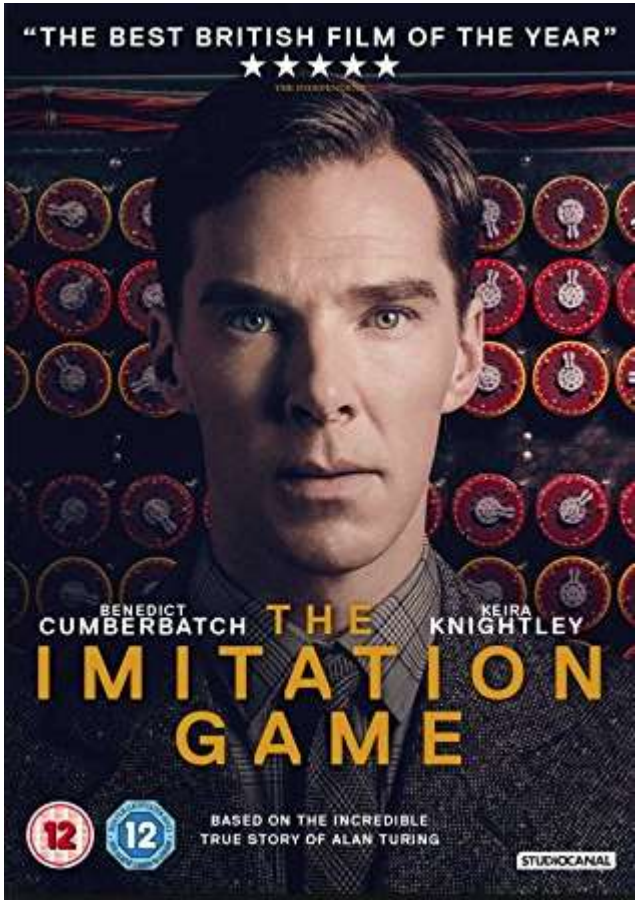
Different in purpose…

Applied math is math with practical use.

Pure math is math done for its own sake.

ALAN TURING


Pure Math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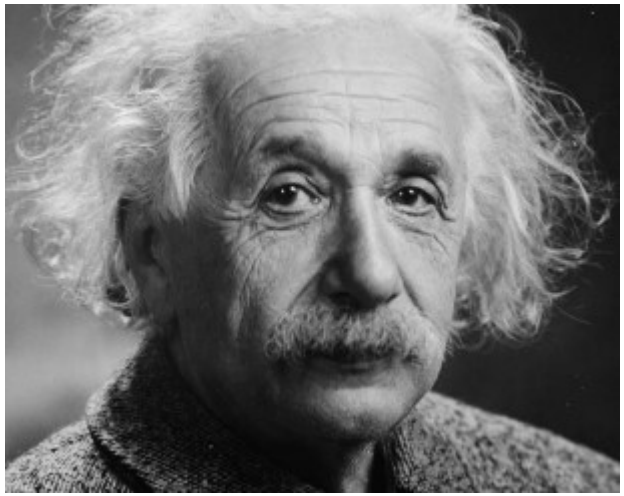
KEIRA KNIGHTLEY
12
12
BASED ON THE INCREDIBLE TRUE STORY OF ALAN TURING
STUDIOCANAL

AND THERE'S EVEN MORE..

Non-Euclidean Geometry

↓

Theory of Relativity



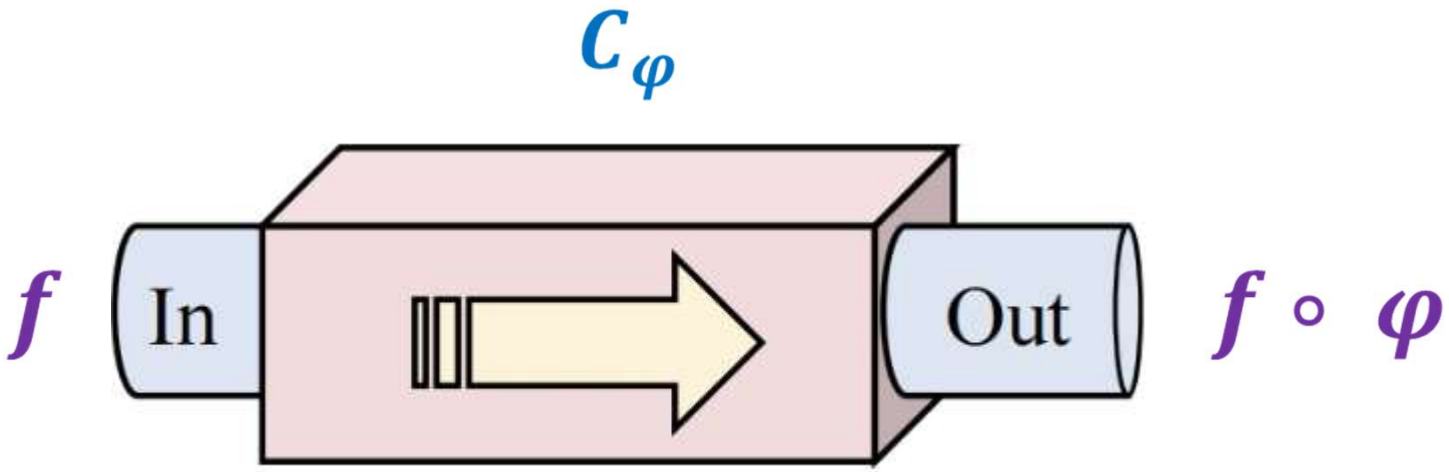
Does Pure Mathematics ALWAYS have Applications?

NO. In fact, in most cases, it does NOT.

"I am a pure mathematician and I am motivated by mathematical questions which are natural and beautiful, rather than by specific applications"

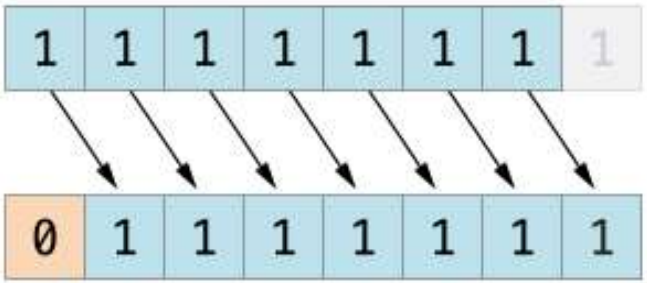
- Professor Ben Green, Waynflete Professor of Pure Mathematics, University of Oxford, United Kingdom.

Composition Operator

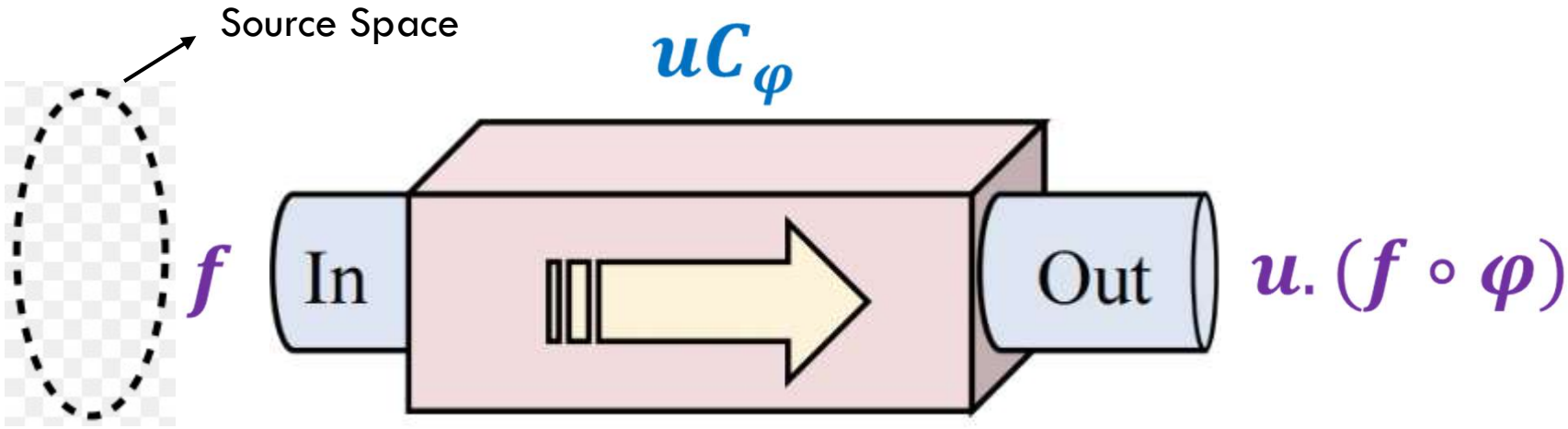


φ: Some initial data

Example: Shift Operator



Weighted Composition Operator



φ , u : **Two sets of initial data**

A Generalization of Composition Operator

WHY STUDY COMP OPERATOR?

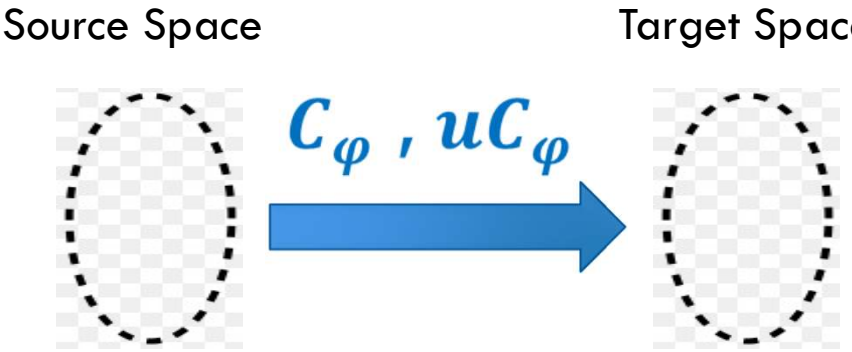
Primary Goal

How does changing the initial data and/or the source space affect the behavior of the operator?

Source Spaces

Source spaces are OFTEN Banach/Hilbert spaces, e.g. L^p spaces, Bergman space, and Hardy space

PROPERTIES WE STUDY



Operator Property	Intuitive Meaning
Bounded	Target space is “Not too big”
Compact	Output signals are “Not too scattered”
Hilbert-Schmidt (HS)	Outputs generated by a particular class of inputs are “Not too big”

HISTORY

All Comp Operators on **Bergman** space are BOUNDED (Littlewood, 1925)

NO unsolved problems found for **single** Comp/Weighted Comp Operators.

What's next?

What about the **sum/difference** of two comp operators?

KNOWN → UNKNOWN

$(C_\varphi - C_\psi)$ is bounded/compact/HS ≡ Two weighted comp operators are bounded/compact/HS. (Choe, Hosokawa, Koo, Saukko, 2011)

$(uC_\varphi - vC_\psi)$ is bounded/compact/HS ≡ Four weighted comp operators are bounded/compact/HS. (Acharyya, Wu, 2017)

ONGOING WORK

Essential Norm

Compact: Output signals are "Not too scattered"

Essential Norm: Some measurement of SCATTEREDNESS

Compact ≡ E.N = 0

Ongoing work

Estimate E.N of

$$uC_{\varphi} - vC_{\psi}$$

Challenge: Lack of symmetry

PRIOR WORK

Published

Acharyya, Soumyadip and Wu, Zhijian. *Compact and Hilbert-Schmidt Differences of Weighted Composition Operators*. Integral Equations and Operator Theory. (2017) 1 – 18

Submitted

Acharyya, Soumyadip and Ferguson, Timothy. *Sums of Weighted Differentiation Composition Operators*. Submitted. (Available on arXiv.org)

Operator Theory in Applications

Quantum Mechanics

Control Theory

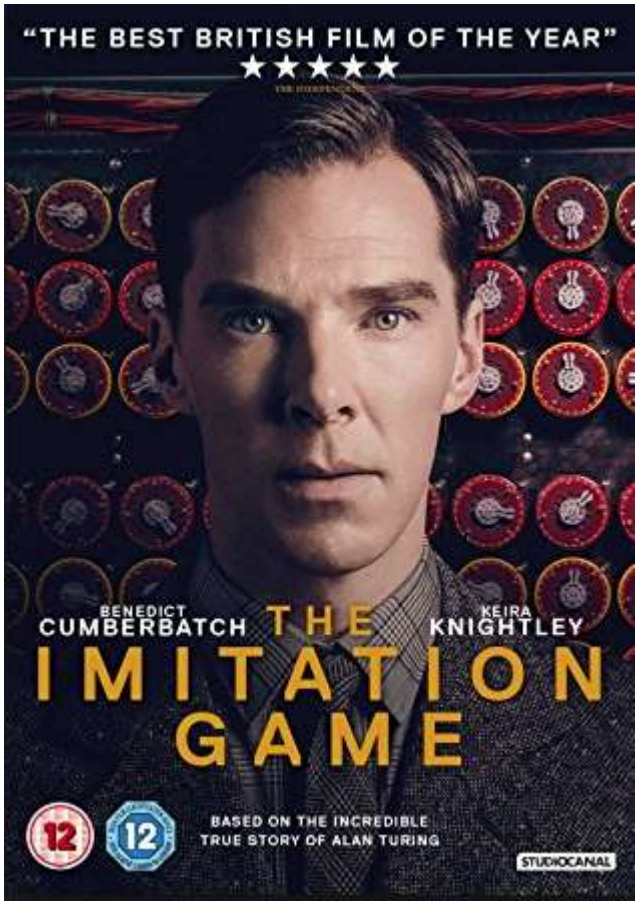
Signal Processing

MATH EVENTS


JOINT
MATHEMATICS
MEETINGS
BALTIMORE • JAN 16–19, 2019
AMERICAN MATHEMATICAL SOCIETY
MATHEMATICAL ASSOCIATION OF AMERICA


Georgia Tech School of Mathematics
College of Sciences
March 23-25, 2018
Southeastern Analysis Meeting
SEAM 34

HW?



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